**A GLANCE AT PROPOSITIONAL LOGIC**

A proposition is the study of logic. It is defined as a declarative statement that is either TRUE or FALSE. The propositional logic is defined recursively which is also known as WFF (Well Formed Formula) or formula, for example:

1. If A is wff then (~A) is a wff.
2. If A and B are formula, then each of following are wff:
   1. (A ᴧ B)
   2. (A v B)
   3. (A→B)
   4. (A↔B)
3. Any wff is obtained only by applying the rules or operator between operands.

It is an easy to identify the incorrect formula such as:

1. (P→(R ᴧ))
2. (P(R ᴧ A)).

The mentioned above formulas are not wff because in second formula ‘P’ truth value is not formulated with (R ᴧ A). The correct formula could be (P→(R ᴧ A)) or other.

We can also eliminate or ignore parentheses by setting up the priorities in increasing order.

|  |  |  |
| --- | --- | --- |
| Priority | Name | Symbol |
| 1. | Negation | ~ |
| 2. | Conjunction | ᴧ |
| 3. | Disjunction | V |
| 4. | Implication | → |
| 5. | If and only if | ↔ |

**MEANING OF PROPOSITIONAL LOGIC**

Please refer this below table to find out the truth value. Consider, if we have one statement then possible interpretations will be 21, if two statements then interpretations will be 22 and so on.

|  |  |  |
| --- | --- | --- |
| S no | Name | Description |
| 1. | Negation | |  |  | | --- | --- | | A | ~A | | T | F | | F | T | |
| 2. | Conjunction | |  |  |  | | --- | --- | --- | | A | B | (AᴧB) | | T | T | T | | T | F | F | | F | T | F | | F | F | F | |
| 3. | Disjunction | |  |  |  | | --- | --- | --- | | A | B | (AvB) | | T | T | T | | T | F | T | | F | T | T | | F | F | F | |
| 4. | Implication | |  |  |  | | --- | --- | --- | | A | B | (A→B) | | T | T | F | | T | F | F | | F | T | T | | F | F | F | |
| 5. | If and only if | |  |  |  | | --- | --- | --- | | A | B | (A↔B) | | T | T | T | | T | F | F | | F | T | F | | F | F | F | |

**EQUIALENT FORMS IN THE PL**

A1 and A2 are logically equal for each following interpretation:

1. If A1 and A2 have same truth values.
2. A1 is true if and only if A2 is true and vice versa.

~(D→F) = D ᴧ ~F

Try to prove it for practice.

|  |  |  |
| --- | --- | --- |
| Propositional Logic Equivalence Table | | |
| S no | Equation | Comment |
| 1. | (E↔G)=(E→G) ᴧ (G→E) |  |
| 2. | (E→G) = ~E v G |  |
| 3. | E v G = G v E | Commutative law |
| 4. | (E v G) v H = E v (G v H) | Associative law |
| 5. | E v (G ᴧ H) = (E v G) ᴧ (E v H) | Distributive law |
| 6. | E v False = E |  |
| 7. | E v True = True |  |
| 8. | ~(~E) = E |  |
| 9. | ~(E v G) = ~E ᴧ ~G | De Morgan’s law |

|  |  |
| --- | --- |
| **NORMAL FORMS:** | Important\*\* |

1. **CNF (Conjunctive Normal Form):** A wff is a conjunctive normal form if and only if ‘F’ has the form ‘F1 ᴧ F2 ᴧ … ᴧ Fn’ n>1 and F1 can be any wff such as (E v G) ᴧ (E ᴧ G), etc. For example:

F: (E v G) ᴧ (E ᴧ T) ᴧ ((E ᴧ G)→A)

1. **DNF (Disjunctive Normal Form):** A wff G is a disjunctive normal form if and only if G has form such as mentioned below :

G: (E ᴧ G) v (E ᴧ T) v ((E ᴧ G)→A)

In order to convert formula into DNF, please follow the simple steps mentioned below:

1. Remove the logical operators ‘↔’,’→’
   1. (E↔G)=(E→G) ᴧ (G→E)
   2. (E→G) = ~E v G
2. Remove ‘~’ if occur consecutively more than once:
   1. ~(~E) = E
   2. ~(E v G) = ~E ᴧ ~G
   3. ~(E ᴧ G) = ~E v ~G
3. Apply distributive law
   1. E v (G ᴧ H) = (E v G) ᴧ (E v H)
   2. E ᴧ (G v H) = (E ᴧ G) v (E ᴧ H)